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Master’s study:

Subject: Econometrics

Field of study: Finance and Accounting

Student’s first name and surname: Linh Nguyen

Student’s Register No: 124112

**ECONOMETRIC PROJECT**

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1. **Introduction about data**

In this report I use the data of 10 company that are running their business in the technology sector. In more detail, I will take the daily close price (P) of them in The Nasdaq Stock Market from (1/1/2018 – 28/02/2023 -> we will have 1298 observation) and the 10 firms include Alphabet Inc. (GOOGL), Dell Technologies Inc (DELL), Apple Inc (AAPL), Tesla Inc (TSLA), IBM (IBM), Cisco Systems Inc (CSCO), HP Inc (HPQ), Intel Corporation (INTC), Meta Platforms Inc (META), NVIDIA Corporation (NVDA).

1. **First exercise – Calculating Rt, Average Sigma annual 100 days, Sigma daily, Sigma annual, Standard deviation of Sigma annual, drawing Histogram**
2. Compute Rate of return (ut) with closing price (P) as the formula below:

|  |  |  |
| --- | --- | --- |
| **Date** | **Close price** | **Ut** |
| 02/01/2018 | 23.13 |  |
| 03/01/2018 | 23.38 | 1.08% |
| 04/01/2018 | 23.79 | 1.74% |
| 05/01/2018 | 23.97 | 0.75% |
| 08/01/2018 | 23.96 | -0.04% |

Pt-1 : close price of the previous date. Apply for 10 firms, the table result of one of them (DELL) (ut  should be formated in percentage %):

1. Compute the moving average of the standard deviation of the previous 90 days of returns, which we will use to calculate Sigma (MA90) as the formula below:

: Average rate of return

N = Number of observations (in this case, 90)

In order to simplify the formula we assume the = 0 and replace N-1 by N. We will have the new Std formula:

Therefore with this Std formula, in practice we can directly use the STDEV.S formula function on the rate of return in excel to calculate it.

Because it’s a moving average we will have σ1, σ2, σ3 ……with σ1 represents sigma of the sample of daily return from the day 1 to the day 90, σ2 represents sigma of the sample of daily return from the day 2 to the day 91, σ3 represents sigma of the sample of daily return from the day 3 to the day 92. The first 90 observations start from 2/1/2018 to 10/05/2018 (The number of observation is still 90 but with different value), it will move until its reach the daily return of the last day (28/02/2023).

1. Transform from daily to annual.

By multiplying the Moving average of daily Standard deviation 90 days above with square root of 250 (number of trading days in a year), we achieve annual value.

1. Calculate the Average of Annual standard deviation moving average 90 days (in 3) and the Standard deviation of Annual standard deviation moving average 90 days (in 3)

The final result will look like the table below (DELL):

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| No. | Date | Price | Ut |  |  |
| ….. | ….. | ….. | ….. |  |  |
| 96 | 18/05/2018 | 22.12 | 5.86% | **Average Sigma** | 37.01% |
| 97 | 21/05/2018 | 22.58 | 2.06% | **Std Dev Sigma** | 13.04% |
| 98 | 22/05/2018 | 22.67 | 0.40% | **DailyStd Dev MA(90)** | **AnnStd Dev MA(90)** |
| 99 | 23/05/2018 | 22.98 | 1.36% | 2.06% | 32.52% |
| 100 | 24/05/2018 | 23.2 | 0.95% | 2.05% | 32.47% |
| 101 | 25/05/2018 | 23.02 | -0.78% | 2.05% | 32.34% |
| 102 | 29/05/2018 | 22.61 | -1.80% | 2.04% | 32.31% |
| 103 | 30/05/2018 | 22.74 | 0.57% | 2.04% | 32.33% |

1. Drawing 04 chart: histogram of Annual standard deviation moving average 90 days, histogram of daily return, line chart of Annual standard deviation moving average 90 days Sigma (MA90) = f (t), line chart of daily return Rt = f (t)

* Prepare the data for drawing histogram of Annual standard deviation moving average 90 days:

Some features need to precalculate including Max, Min value of Annual standard deviation moving average 90 days, Bin width ((Max – Min)/the range of values into a series of bin (in this case is 42)).

**Bin value**: We will have 42 bins, the first bin equal to Min value above, the second equal to the first bin + Bin width above.

**Frequency:** use Frequency function in excel to find out how many value will fall into each bin.

**Relative Frequency:** calculate percentage of data points that fall within each bin by taking bin value divide by total number of Annual standard deviation moving average 90 days (in this case is 1200).

**Cumulated Frequency**: the first cumulated frequency equal to the first value of relative frequency, the second cumulated frequency equal to the first cumulated frequency add the second relative frequency.

The data for histogram will look like below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Max** | 74.7% |  |  |  |
| **Min** | 15.6% |  |  |  |
| **Bin Width** | 1.48% |  |  |  |
|  |  |  |  |  |
| **Histogram Sigma MA(90)** | | | | |
| Bin No | Bin Value | Freq | Rel Freq | Cum Freq |
| 1 | 15.6% | 1 | 0.1% | 0.1% |
| 2 | 17.1% | 0 | 0.0% | 0.1% |
| 3 | 18.6% | 4 | 0.3% | 0.4% |
| 4 | 20.0% | 12 | 1.0% | 1.4% |

* Applying those step to draw histogram of daily return, we have the table below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Max** | 13.5% |  |  |  |
| **Min** | -24.4% |  |  |  |
| **Bin Width** | 0.95% |  |  |  |
|  |  |  |  |  |
| **Histogram Rt** | | | | |
| Bin No | Bin Value | Freq | Rel Freq | Cum Freq |
| 1 | -24.4% | 1 | 0.08% | 0.08% |
| 2 | -23.4% | 0 | 0.00% | 0.08% |
| 3 | -22.5% | 0 | 0.00% | 0.08% |
| 4 | -21.5% | 0 | 0.00% | 0.08% |

The 04 graphs of DELL company:

1. **Second exercise – GARCH model**

Build up GARCH (1,1) model using Maximum Likelihood to predict volatility of return on stock price of 10 firms.

1. In GARCH model we have 03 parameters: alpha (, beta (, gamma (. The sum of those parameters must be equal to 1. We will use those 03 parameters to compute the long-run average variance rate as the formula below:

We set with VL stand for long run daily variance => the formula change into:

With sum of 03 parameters equal to 1 => we have the formula:

Before running solver we assume the following initial values for , , as following:

Base on those initial value and the formula , we calculate (long run daily variance) , then taking square root of to obtain Long run daily Std, then by multiplying the Long run daily Std with square root of 250 (number of trading days in a year), we achieve its annual value.

The result for parameters before running solver (DELL):

|  |  |  |
| --- | --- | --- |
|  | **GARCH Model Estimated Parameters (α,β,ɤ)** | |
|  |
| Initial values | Max Likelihood of Sum[-ln(Vt)-Rt^2/Vt)] |  |
|  | ϒ | 0.05 |
| 0.5 | α | 0.5 |
| 0.45 | β | 0.45 |
| 0.0001 | ω | 0.0001 |
|  |  |  |
|  | ϒ+b+y=1 | 1 |
|  | longrun daily variance | 0.002 |
|  | longrun daily std deviation | 4.47% |
|  | longrun annual std deviation | 70.68% |

1. We use maximum likelihood method in this case instead of regular regression cause in this case, our data (the volatility of the rate of return) tends to fluctuate over time and in traditional regression, it is often assumed that the errors (or residuals) have constant volatility. In maximum likelihood methods we choose parameters that maximize the probability of observing the given rate of return data, in other words, choose parameter that maximize the likelihood function below, this function measure the probability of obtaining the observed data:

* To obtain the result for the function, firstly, we need to calculate the ui and v or (variance of Ut) . The t is calculated the same as in I. As for the Variance of Ut, the first volatility value equal to power 2 of the first t. the second variance value is calculated as the fomula:

After achieve the value of ui and Rt we can compute the likelihood function. The result will be look like this (DELL):

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Closing price** | **Daily return** |  |  | **Annual Std before GARCH model** |
| 02/01/2018 | 23.13 |  |  |  |  |
| 03/01/2018 | 23.38 | 1.075% |  |  |  |
| 04/01/2018 | 23.79 | 1.738% | 0.012% |  | 27.5% |
| 05/01/2018 | 23.97 | 0.754% | 0.030% | 7.91 | 11.9% |
| 08/01/2018 | 23.96 | -0.042% | 0.026% | 8.24 | 0.7% |

(Note: for calculating the Annual Std before GARCH model we do the same as in I: multiply square root of 250 with the square root of daily return)

1. After the step 2 we should achieve the Sum of likelihood function. Next step, we use the Solver function for set a constrain for parameters (γ, α, β). As we have already mentioned above those parameters should be settled in the range from 0 to 1. And the sum of them must be equal to 1.

The constrain input into Solver should be like this:

0≤ γ ≤1

0≤ α ≤1

0≤ β ≤1

γ+α+β=1

After running the Solver, we will obtain a new set of parameters that maximize the Sum of likelihood function.

The result for DELL:

|  |  |  |
| --- | --- | --- |
|  | **GARCH Model Estimated Parameters (α,β,ɤ)** | |
|  |
| Initial values | Max Likelihood of Sum[-ln(Vt)-Rt^2/Vt)] | 8454.71 |
|  | ϒ | 0.149 |
| 0.5 | α | 0.088 |
| 0.45 | β | 0.762 |
| 0.0001 | ω | 0.000087 |
|  |  |  |
|  | ϒ+b+y=1 | 1 |
|  | longrun daily variance | 0.06% |
|  | longrun daily std deviation | 2.42% |
|  | longrun annual std deviation | 38.23% |

1. Recalculte feature in step 2 with this new set of parameters. The result will be like below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Closing price** | **Daily return** |  |  | **Annual Std before GARCH model** |
| 02/01/2018 | 23.13 |  |  |  |  |
| 03/01/2018 | 23.38 | 1.075% |  |  |  |
| 04/01/2018 | 23.79 | 1.738% | 0.012% |  | 27.5% |
| 05/01/2018 | 23.97 | 0.754% | 0.020% | 8.23 | 11.9% |
| 08/01/2018 | 23.96 | -0.042% | 0.025% | 8.31 | 0.7% |

We also added 03 new features: GARCH Modeled Daily Std, GARCH Modeled Annual Std Dev and Long Run Annual Std. Computing the daily Std and Annual Std is the same as in I. As for Long run annual Std, we simply take Long run annual Std in the parameter table in step 3. The table result will be like this:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Date** | **Closing price** | **Daily return=u** |  |  | **Before-GARCH Std Dev** | **GARCH Model Daily Std Dev** | **GARCH Model annual Std Dev** | **Long run annual std dev** |
| 02/01/2018 | 23.13 |  |  |  |  |  |  |  |
| 03/01/2018 | 23.38 | 1.075% |  |  |  |  |  |  |
| 04/01/2018 | 23.79 | 1.738% | 0.012% |  | 27.5% | 1.1% | 17.0% | 38.23% |
| 05/01/2018 | 23.97 | 0.754% | 0.020% | 8.23 | 11.9% | 1.4% | 22.5% | 38.23% |
| 08/01/2018 | 23.96 | -0.042% | 0.025% | 8.31 | 0.7% | 1.6% | 24.8% | 38.23% |

X

1. Forecased Standard Deviation over 300 days ahead:

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  Description automatically generatedEstimating Variance by applying the formula below:

VL: longrun daily variance

: variance n (the last value of the variance of the rate of return – historical data)

* Base on the estimated variance, we estimate GARCH Modeled Daily Std by taking the square root of estimated variance above. Then estimate sigma t (GARCH Model annual Std Dev) by multiplying estimated GARCH Model Daily Std with the square root of 250. The sigma L (Long run annual Std) is the same.

The estimated result will be look like this (DELL):

**Forecased Standard Deviation over 300 days ahead**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. |  | Estimate GARCH Model Daily Std | Sigma t (GARCH Model annual Std) | Sigma L (Long run annual Std) |
| 1 | 0.044% | 2.1% | 33.1% | 38.23% |
| 2 | 0.046% | 2.1% | 33.9% | 38.23% |
| 3 | 0.048% | 2.2% | 34.6% | 38.23% |
| … | … | … | … | … |

1. Find adjusted sigma n (adjusted Std n). As we observe via the chart in the step 4, the volatility of return on the stock price is fluctuated around some certain value called the long-run volatility (or long run annual Std or sigma L). The greater the volatility, the higher the risk. By calculating adjusted sigma n we know when (in the future) the volatility of the return will return to the long run annual Std.

(Note: The higher the sum of *α + β* the slower the volatility of the return will return to the long run annual Std). The process to find adjusted sigma n:

* Determine the sigma n. Sigma n is the last value of the volatility of the rate of return (historical volatity data).
* Determine the lowest point of volatility by observing the chart or simply compute Min function excel on GARCH model annual Std.
* From that min value (lowest point) we calculate adjusted sigma daily by deviding it by square root of 250.
* Then, we power 2 the adjusted sigma daily to obtain adjusted variance.
* Replace adjusted variance to the last old one in column.

Before we start to adjust sigma n, we need to ganerate a Check error indicator to check when (in the future) the percentage difference between sigma t and sigma L is less than 0.5%, in other word, they start to approach each other (X point in Estimated sigma graph above). If it > 0.5% the indicator will return 1. If it isn’t, the indicator will return 0. By using If function excel we can satisfy all those conditions.

After compute the estimated GARCH model we fill out the table below:

| No. | Firms | γ | α | β | α+β+ɤ=1 | Long Run Annual Std Dev | Sigma(n) Annual | Time to return to sigma L | Sigma N adjusted | Time to Return to the Adjusted Sigma N (Tk) | Historical Sigma less Adjusted Sigma | Historical Time less Adjusted Time |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | AAPL | 0.045 | 0.119 | 0.836 | 1.0000 | 33.42% | 25.64% | 81 | 0.270% | 100 | 25.37% | -19 |
| 2 | TSLA | 0.061 | 0.076 | 0.863 | 1.0000 | 65.30% | 64.96% | 0 | 16.27% | 73 | 48.69% | -73 |
| 3 | IBM | 0.086 | 0.095 | 0.819 | 1.0000 | 26.48% | 21.18% | 39 | 18.89% | 44 | 2.29% | -5 |
| 4 | CSCO | 0.195 | 0.302 | 0.503 | 1.0000 | 29.80% | 22.72% | 18 | 12.59% | 21 | 10.12% | -3 |
| 5 | GOOGL | 0.049 | 0.081 | 0.870 | 1.0000 | 31.46% | 35.15% | 0 | 20.45% | 81 | 14.70% | -81 |
| 6 | HPQ | 0.072 | 0.071 | 0.857 | 1.0000 | 37.01% | 31.25% | 45 | 3.62% | 62 | 27.62% | -17 |
| 7 | INTC | 0.234 | 0.108 | 0.658 | 1.0000 | 36.55% | 35.35% | 8 | 30.40% | 13 | 4.95% | -5 |
| 8 | META | 0.077 | 0.307 | 0.616 | 1.0000 | 64.40% | 31.37% | 54 | 28.07% | 55 | 3.29% | -1 |
| 9 | NVDA | 0.025 | 0.122 | 0.853 | 1.0000 | 57.89% | 76.09% | 0 | 28.15% | 171 | 47.94% | -171 |
| 10 | DELL | 0.149 | 0.089 | 0.762 | 1.0000 | 38.23% | 17.00% | 21 | 17.00% | 27 | 0.00% | -6 |

(Note:

* Time to return to sigma L (T) – is the time point before we adjust sigma n and when it start to equal to 0, we need to check the value of Check error indicator.
* Time to return to adjusted sigma n (Tk) – After we compute the new variance from the adjusted sigma n => we replace it into the last variance (as mentioned above). After the replacement the T will change. And the new T is Tk.

1. **Cholesky decomposition and Monte Carlo simulation**

We use Cholesky decomposition and Monte Carlo simulation to minimize the variance of the portfolio. In order to compute the Cholesky matrix into portfolio including 10 firms we follow the procedure below:

* Firstly, we need to find the correlation coefficient of 10 firms, we run the correlation coefficient function in excel and generate the result table as below:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **CORRELATION MATRX** | | | | | | | | | |
|  | TSLA | AAPL | DELL | NVDA | META | INTC | HPQ | IBM | GOOGL | CSCO |
| TSLA | 1.0000 |  |  |  |  |  |  |  |  |  |
| AAPL | 0.3877 | 1.0000 |  |  |  |  |  |  |  |  |
| DELL | 0.3158 | 0.3983 | 1.0000 |  |  |  |  |  |  |  |
| MSFT | 0.4889 | 0.5371 | 0.4771 | 1.0000 |  |  |  |  |  |  |
| META | 0.3324 | 0.4850 | 0.3537 | 0.5461 | 1.0000 |  |  |  |  |  |
| INTC | 0.3678 | 0.4870 | 0.4806 | 0.5999 | 0.4577 | 1.0000 |  |  |  |  |
| HPQ | 0.3311 | 0.4281 | 0.5979 | 0.5082 | 0.3740 | 0.5276 | 1.0000 |  |  |  |
| IBM | 0.2074 | 0.3711 | 0.4706 | 0.3610 | 0.2768 | 0.5023 | 0.5240 | 1.0000 |  |  |
| GOOGL | 0.4108 | 0.5734 | 0.4796 | 0.6590 | 0.6643 | 0.5764 | 0.5087 | 0.4519 | 1.0000 |  |
| CSCO | 0.2822 | 0.4905 | 0.5271 | 0.5148 | 0.4075 | 0.5765 | 0.5509 | 0.5907 | 0.5708 | 1.0000 |

Base on the Correlation matrix we can simply run CHOL function in excel to obtain Cholesky matrix, as table below:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Cholesky Matrix** | | | | | | | | | |
|  | TSLA | AAPL | DELL | NVDA | META | INTC | HPQ | IBM | GOOGL | CSCO |
| TSLA | 1.000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| AAPL | 0.388 | 0.922 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DELL | 0.316 | 0.299 | 0.900 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| MSFT | 0.489 | 0.377 | 0.233 | 0.751 | 0 | 0 | 0 | 0 | 0 | 0 |
| META | 0.332 | 0.386 | 0.148 | 0.271 | 0.803 | 0 | 0 | 0 | 0 | 0 |
| INTC | 0.368 | 0.374 | 0.281 | 0.285 | 0.090 | 0.747 | 0 | 0 | 0 | 0 |
| HPQ | 0.331 | 0.325 | 0.440 | 0.161 | 0.037 | 0.150 | 0.736 | 0 | 0 | 0 |
| IBM | 0.207 | 0.315 | 0.345 | 0.080 | 0.017 | 0.251 | 0.204 | 0.792 | 0 | 0 |
| GOOGL | 0.411 | 0.449 | 0.239 | 0.310 | 0.292 | 0.101 | 0.062 | 0.094 | 0.606 | 0 |
| CSCO | 0.282 | 0.413 | 0.349 | 0.186 | 0.065 | 0.217 | 0.142 | 0.230 | 0.093 | 0.678 |

However, we can obtain the values for this table with the below procedure (we will work on a smaller sample but the logic is still the same):

* The Correlation matrix and Cholesky matrix of 6 firms is constructed and illustrated as below:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Cholesky Matrix** | | | | |  |
|  | TSLA (1) | AAPL (2) | DELL (3) | NVDA (4) | META (5) | INTC (6) |
| TSLA (1) | A11 | 0 | 0 | 0 | 0 | 0 |
| AAPL (2) | A21 | A22 | 0 | 0 | 0 | 0 |
| DELL (3) | A31 | A32 | A33 | 0 | 0 | 0 |
| MSFT (4) | A41 | A42 | A43 | A44 | 0 | 0 |
| META (5) | A51 | A52 | A53 | A54 | A55 | 0 |
| INTC (6) | A61 | A62 | A63 | A64 | A65 | A66 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Correlation Matrix** | | | | |  |
|  | TSLA (1) | AAPL (2) | DELL (3) | NVDA (4) | META (5) | INTC (6) |
| TSLA (1) | 1 |  |  |  |  |  |
| AAPL (2) | p21 | 1 |  |  |  |  |
| DELL (3) | p31 | p32 | 1 |  |  |  |
| MSFT (4) | p41 | p42 | p43 | 1 |  |  |
| META (5) | p51 | p52 | p53 | p54 | 1 |  |
| INTC (6) | p61 | p62 | p63 | p64 | p65 | 1 |

* Each value of a Correlation Matrix will be contructed as formula below:
* Line 1: correlation of the same companny equal to 1. => A11 = 1
* Line 2: p21 = A21\* A11 => A21

p22 = 1 = A221\* A222 => A22

* Line 3: p31 = A31 \* A11 => A31

p32 = A31 \* A21 + A32 \* A22 => A32

p33 = 1 = A231\* A232 \* A233 = 1=> A33

* Line 4: p41 = A41 \* A11 => A41

p42 = A41 \* A21 + A42 \* A22 => A42

p43 = A41 \* A31 + A42 \* A32 + A43 \* A33 => A43

p44 = A241  + A242 + A243 + A243  = 1 => A44

* Line 5: p51 = A51 \* A11 => A51

p52 = A51 \* A21 + A52 \* A22 => A52

p53 = A51 \* A31 + A52 \* A32 + A53 \* A33 => A53

p54 = A51 \* A41 + A52 \* A42 + A53 \* A43 + A54 \* A43=> A54

p55 = A251  + A252 + A253 + A254 + A255 = 1 => A55

* Line 6: p61 = A61 \* A11 => A61

p62 = A61 \* A21 + A62 \* A22 => A62

p63 = A61 \* A31 + A62 \* A32 + A63 \* A33 => A63

p64 = A61 \* A41 + A62 \* A42 + A63 \* A43 + A64 \* A44=> A64

p65 = A61 \* A51 + A62 \* A52 + A63 \* A53 + A64 \* A54 + A65 \* A55=> A65

p66 = A261  + A262 + A263 + A264 + A265 + A266 = 1 => A66

(Note: we don’t need to remember the formula, the logic to form those formula is to find the common element in the subscript parts. For example, regarding p65, we have the subscript element equal to 65 => all the subscript parts of the multiplication should be equal to 65. In detail, p65 = A61 \* A51 + A62 \* A52 + A63 \* A53 + A64 \* A54 + A65 \* A55)

* Each line of the Cholesky metrix attached with the formula above:
* Line 1: e1 = A11 \* X1
* Line 2: e2 = A21 \* X1 + A22 \* X2
* Line 3: e3 = A31 \* X1 + A32 \* X2 + A33 \* X3
* Line 4: e4 = A41 \* X1 + A42 \* X2 + A43 \* X3 + A44 \* X5
* Line 5: e5 = A51 \* X1 + A52 \* X2 + A53 \* X3 + A54 \* X4 + A55 \* X5
* Line 6: e6 = A61 \* X1 + A62 \* X2 + A63 \* X3 + A64 \* X4 + A65 \* X5 + A66 \* X6

X: random standard normal distribution

* Secondly, we generate 1000 random numbers from a standard normal distribution by running RAND() function and NORMSINV() function in Excel. Those scenarios are represented in letter X as mentioned above with a mean of 0 and a standard deviation of 1. We run RAND() to generate a random number between 0 and 1. NORMSINV() takes that random number and calculates its inverse cumulative distribution for a standard normal distribution. The result table (the result will be different since we use random function):

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **x1** | **x2** | **x3** | **x4** | **x5** | **x6** | **x7** | **x8** | **x9** | **x10** |
| 0.800 | 1.212 | -0.529 | -1.381 | -0.666 | -0.968 | 0.682 | 0.441 | -1.152 | 0.303 |
| 0.155 | -0.456 | -1.299 | 0.472 | -1.569 | 0.329 | -0.016 | 0.695 | 0.750 | -0.411 |
| 0.584 | 0.817 | -0.611 | 0.473 | -0.706 | 0.462 | -1.238 | -1.409 | 1.436 | -0.039 |
| -0.864 | -0.112 | -0.026 | -0.040 | -0.305 | -1.706 | 0.505 | 0.342 | -1.120 | 0.184 |
| 1.708 | -0.190 | 0.147 | 0.598 | -1.196 | 0.304 | 1.825 | 0.063 | 0.127 | -0.168 |

* Third, multiply the lower triangular matrix by standard normal random (in step 2) to obtain a vector of correlated random (e1, e2, e3…. e10). We use formulas in step 1 to calculate e. The result table:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **e1** | **e2** | **e3** | **e4** | **e5** | **e6** | **e7** | **e8** | **e9** | **e10** |
| -0.470 | -0.019 | 1.490 | -0.140 | -0.165 | 0.057 | 1.041 | 0.356 | -0.304 | 0.612 |
| -1.431 | 0.069 | 0.678 | 0.040 | 0.236 | -0.672 | 0.062 | -0.659 | 0.448 | 0.481 |
| 0.806 | 0.382 | 1.919 | 1.723 | -0.298 | 1.494 | 1.647 | -0.211 | 1.077 | 0.808 |
| 1.460 | -0.241 | 0.431 | -0.328 | 0.248 | 0.089 | 0.462 | -0.738 | -0.422 | -0.326 |
| 0.427 | -0.598 | 0.598 | 0.253 | 0.162 | -0.319 | -0.091 | -0.605 | -0.052 | -0.978 |

* Forth, we have w1, w2, w3 …w10 stand for proportion of each company in the portfolio. In the initial, we can set an equal proportion (0.01) for each company (sum of proportion must be equal to 1)
* Next, calculate the value of each company (A1, A2 … A10) in portfolio by multiplying its proportion with the budget portfolio (in this case, the budget portfolio is 1,000,000)
* Then base on those result above we calculate value of each company (V1, V2, …. V10) in the future taking into account the forecasted volatility (forecasted sigma 300 days GARCH model) by using belowed formulas:

V1 = A1 + A1 \* forecasted daily sigma GARCH model \* e1

V2 = A2 + A2 \* forecasted daily sigma GARCH model \* e2

V3 = A3 + A3 \* forecasted daily sigma GARCH model \* e3

…

V10 = A10 + A10 \* forecasted daily sigma GARCH model \* e10

Forecasted daily sigma GARCH model: this value have been calculated in II (GARCH model), we use Vlookup function to look up for corresponding daily sigma with different T (date).

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **V1** | **V2** | **V3** | **V4** | **V5** | **V6** | **V7** | **V8** | **V9** | **V10** | **V** |
| 240893 | 86376 | 105248 | 123723 | 160232 | 116448 | 85328 | 50579 | 39611 | 34674 | 1043111 |
| 228460 | 84693 | 99161 | 118623 | 161547 | 113663 | 84765 | 51067 | 39471 | 34733 | 1016182 |
| 217810 | 84864 | 100686 | 119681 | 164231 | 112851 | 82991 | 50835 | 39010 | 35222 | 1008183 |
| 222756 | 85540 | 100945 | 122522 | 163070 | 110903 | 82094 | 49859 | 40732 | 34835 | 1013257 |

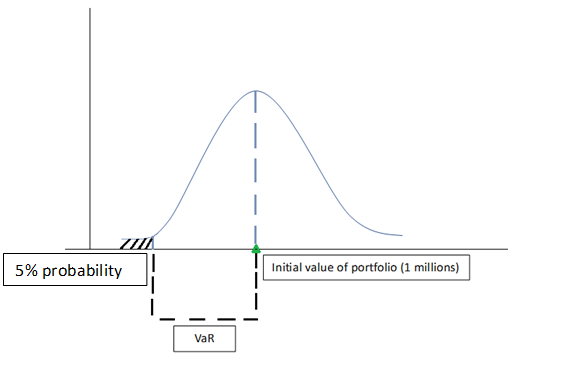
(V (value of 10 firms in portfolio or value of value of = V1 + V2 + V3 + …+V10)

* Compute the function PERCENTILE (V in the previous step, 0.05) to calculates the 5th percentile of the value of our portfolio at a 5% probability (95% confidence level). In other words, calculating the value below which 5% of the data points lie.

|  |  |
| --- | --- |
| **PERCENTILE VaR threshold** | 968,938 |
| **VaR** | 31,062 |

* Next, calculate VaR (the maximum loss in the value at a certain confidence level), VaR is determined by subtracting the portfolio value at the chosen percentile threshold from the portfolio value (result from the PERCENTILE calculation in previous step). This represents the estimated potential loss at the 95% confidence level. In this case, my results as:

These values mean, with the portfolio worth 1,000,000 dollars, there is 5% probability that the losses will not exceed 31,062 (with 95% confidence level). We can understand VaR through the graph chart below:



VaR

Return

Loss

Gain

968,938

31,062

* Finally, set a budget constraint (set constraint for proportional value of each firm) at that VaR is minimum. The constraints is set up as below:

w1 <= 1

w2 <= 1

w3 <= 1

…..

w10 <= 1

w = 1

After setting the constraint, the result table for w, A and VaR will change:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **PV** | **1,000,000** |  |  |  |  |  |  |  |  |
| **TSLA** | **AAPL** | **DELL** | **NVDA** | **META** | **INTC** | **HPQ** | **IBM** | **GOOGL** | **CSCO** |
| **w1** | **w2** | **w3** | **w4** | **w5** | **w6** | **w7** | **w8** | **w9** | **w10** |
| 0.111 | 0.100 | 0.098 | 0.098 | 0.098 | 0.099 | 0.100 | 0.099 | 0.099 | 0.098 |
|  |  |  |  |  |  |  |  |  |  |
| **A1** | **A2** | **A3** | **A4** | **A5** | **A6** | **A7** | **A8** | **A9** | **A10** |
| 110,992 | 99,528 | 97,500 | 98,494 | 98,387 | 99,269 | 99,508 | 99,205 | 98,732 | 98,385 |

* We prepare data to draw histogram for Portfolio value, the procedure for preparing is the same as mentioned in I. Bin value is calculated from Max, Min of VaR.

|  |  |
| --- | --- |
| **V** |  |
| 941,666 | **Min** |
| 1,062,668 | **Max** |
| 3,025 | **Bin Width** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Bin No** | **Bin Value** | **Freq** | **Rel Freq** | **Cum Freq** |
| 1 | 941,666 | 1 | 0.1% | 0.1% |
| 2 | 944,691 | 1 | 0.1% | 0.2% |
| 3 | 947,716 | 3 | 0.3% | 0.5% |
| 4 | 950,741 | 1 | 0.1% | 0.6% |
| 5 | 953,766 | 2 | 0.2% | 0.8% |
| 6 | 956,791 | 4 | 0.4% | 1.2% |

Additional:

Comparison of VaR and the structure of forecasted portfolio for 8 cases: T=10 days; T=20 days; T=50 days; T=100 days; T= 150 days; T=200 days; T=250 days; T=300 days. VaR and structures of the portfolio for this 8 cases

| T | VaR |
| --- | --- |
| T = 10 days | 24,428 |
| T = 20 days | 27,420 |
| T = 50 days | 29,872 |
| T = 100 days | 30,828 |
| T = 150 days | 32,156 |
| T = 200 days | 29,385 |
| T = 250 days | 30,737 |
| T = 300 days | 31,152 |

The structure of portfolio won’t change. Value of our portfolio is 1,000,000 dollars, at given T (time) with the same 5% probability (95% confidence level) we have different VaR and we can interpret the result as below:

T = 10 days: The VaR estimate is 24,428. This means that with a 95% confidence level, there is a 5% probability that our portfolio will not lose more than $24,428.

T = 20 days: The VaR estimate is 27,420. This indicates that there is a 5% chance that our portfolio will not lose more than than $27,420 (with a 95% confidence level).

T = 50 days: The VaR estimate is 29,872. This indicates that there is a 5% chance that our portfolio will not lose more than than $29,872 (with a 95% confidence level).

T = 100 days: The VaR estimate is 30,828. This indicates that there is a 5% chance that our portfolio will not lose more than than $30,828 (with a 95% confidence level).

T = 150 days: The VaR estimate is 32,156. This indicates that there is a 5% chance that our portfolio will not lose more than than $32,156 (with a 95% confidence level).

T = 200 days: The VaR estimate is 29,385. This indicates that there is a 5% chance that our portfolio will not lose more than than $29,385 (with a 95% confidence level).

The same explaination for the remaining time point.

Suppose we have an economic capital for our portfolio is equal to 29,497 we only have ability to cover the losses in day 10, 20, 50 because at those time point VaR is less than the economic value.